Dr. Sam **Name: ________________ ________**
October 20, 2020 **Last First**
Math 116, TEST 3
9:25 am

This is not multiple choice questions. Just mere answer is not acceptable. Each Problem should have enough EXPLANATION.

1. **Verify the following**

   i. $\cos^4 A - \sin^4 A = 2\cos^2 A - 1$

   ii. $(1 - \cot A)^2 = \csc^2 A - 2\cot A$

2. **Subtitle x = 2 Sec y and simplify** $\sqrt{x^2 - 4}$

3. **Find the exact value of**

   i. $\sin(\pi/12)$

   ii. $\cos(7\pi/12)\cos(3\pi/12) + \sin(7\pi/12)\sin(3\pi/12)$

   iii. **if tan A = 3/4 and A in quadrant III, and sin B = 5/13, B in quadrant II,**

   find cos (A + B)

4. **If cos A = -5/13, A in quadrant II, find the exact value of**

   i. $\sin 2A$

   ii. $\cos 2A$

5. **if sin A = 3/5 and A in quadrant II, find the exact value of**

   i. $\cos(A/2)$

   ii. $\sin(A/2)$

NO Calculator Value For Any Problem

Begin with the left side of the original equation and use ...

$$\sin x \cos\frac{x}{2} = 2\sin\frac{x}{2}\cos\frac{x}{2}\cos\frac{x}{2}$$ Replace $\sin x$ with $2\sin\frac{x}{2}\cos\frac{x}{2}$.

$$= 2\sin\frac{x}{2}\cos^2\frac{x}{2}$$

$$= 2\left(\sin\frac{x}{2}\right)\left(\frac{1+\cos\left[2\left(\frac{x}{2}\right)\right]}{2}\right)$$ Power-reducing formula for cosine

$$= \sin\frac{x}{2}(1+\cos x)$$ $\cos\left[2\left(\frac{x}{2}\right)\right] = \cos x$; simplify.

Because the left side is identical to the right side of the equation, the identity is verified.

Practice Problem 11 Verify the identity $\sin\frac{x}{2}\sin x = \cos\frac{x}{2}(1-\cos x)$.

SUMMARY OF MAIN FACTS

Difference and Sum Formulas

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$
$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$
$$\tan(u-v) = \frac{\tan u - \tan v}{1+\tan u\tan v}$$
$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$
$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$
$$\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u\tan v}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2}-x\right) = \cos x \qquad \cos\left(\frac{\pi}{2}-x\right) = \sin x \qquad \tan\left(\frac{\pi}{2}-x\right) = \cot x$$
$$\csc\left(\frac{\pi}{2}-x\right) = \sec x \qquad \sec\left(\frac{\pi}{2}-x\right) = \csc x \qquad \cot\left(\frac{\pi}{2}-x\right) = \tan x$$

Double-Angle Formulas

$$\sin 2x = 2\sin x\cos x$$
$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = 2\cos^2 x - 1$$
$$\cos 2x = 1 - 2\sin^2 x$$

Power-Reducing Formulas

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2} \qquad \tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

The sign + or − depends on the quadrant in which $\frac{\theta}{2}$ lies.